

SPC

LESSON: Sigma Levels - Homework SOLUTIONS

1. This problem's information is from the article **1 in 6 Uber and Lyft Cars Have Open Safety Recalls** written by Ryan Felton (5/29/19) and appearing in a recent edition of Consumer Reports. The following paragraph is taken directly from this source:

Of the **93,958 vehicle identification numbers** (VINs) associated with ride-hailing vehicles in New York City and King County, Wash., that CR examined, **15,175 had one or more open safety recalls**. (Because ride-hailing drivers can work for more than one company, we grouped results together for vehicles associated with Uber, Lyft, and, for New York, smaller competitors Juno and Via.)

Assuming **no shift** in the process mean over time, determine the **Long Term Sigma Level** associated with the number of ride-hailing vehicles in these areas that have one or more open safety recalls.

Instructions for submitting your solution:

- Give your answer correctly rounded to 3 decimal places.
- Copy and paste your graphics from Minitab or your expressions from your Maple worksheet either electronically or using scissors & tape.
- Clearly indicate your final answer by circling it, highlighting it, starring it, etc.
- If you're using a word processor, such as Microsoft Word, reduce the size of large graphics and remove multiple lines of empty space. In other words, reduce the amount of paper that you submit (if possible).

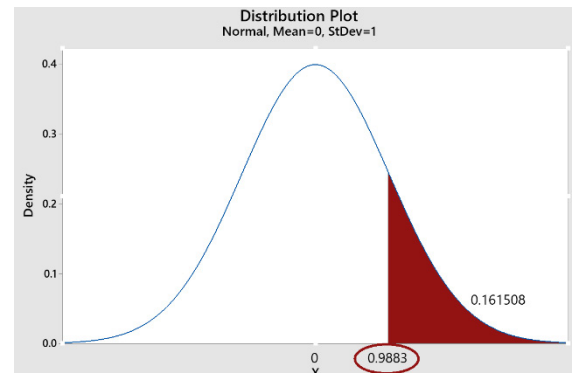
Solution: Since we are told that 15,175 vehicles had one or more open safety recalls out of 93,958 total vehicles, then the proportion defective is: ~ 0.1615083335 ($15175 / 93958$). By using the Probability Distribution Plot function in Minitab, we can see that the Sigma Level corresponding to the proportion of defectives is **$\sim 0.988\sigma$** .

Minitab desktop (20 or higher):

1. Choose **Graph > Probability Distribution Plot**.
2. Choose **View Probability**.
3. Click **Shaded Area**.
4. In **Probability**, enter 0.161508.
5. Click **OK**.

Minitab web app:

1. Choose **Graph > Probability Distribution Plot**.
2. Under **One Curve**, choose **View Probability**.
3. Click **Options**.
4. In **Probability**, enter 0.161508.
5. Click **OK** in each dialog box.



2. Circle the correct answer True or False for the following statements. You do not need to show your work for these.

- a. Doubling any positive Long Term Sigma Level (e.g. going from 2σ to 4σ) will always cut the number of defects per million by at least a half.

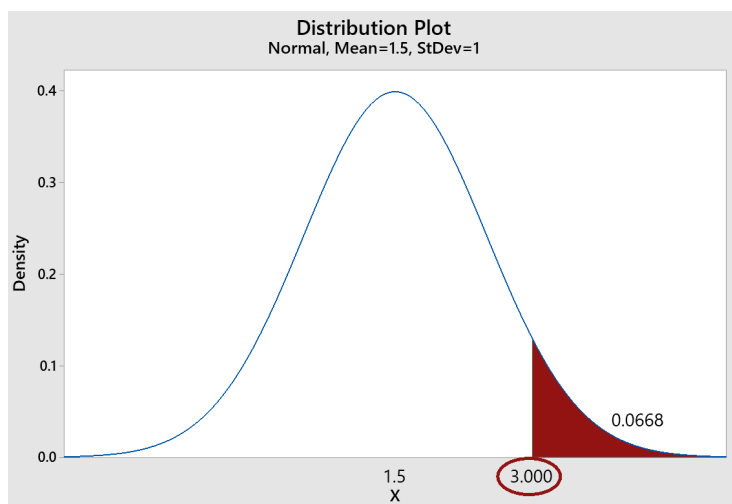
True **False**

Solution: See the Sigma Conversion Chart in the lesson notes. Although this is true for “larger” Sigma Levels, such as 2σ (22,750) to 4σ (32), it is not true for some smaller ones, such as 0.1σ (460,172) to 0.2σ (420,740).

- b. The DLE Corporation operates at a yield of 93.32% (non-defective), meaning 6.68% of its products is defective. This corporation is operating at a Short Term Sigma Level of approximately 3σ .

True False

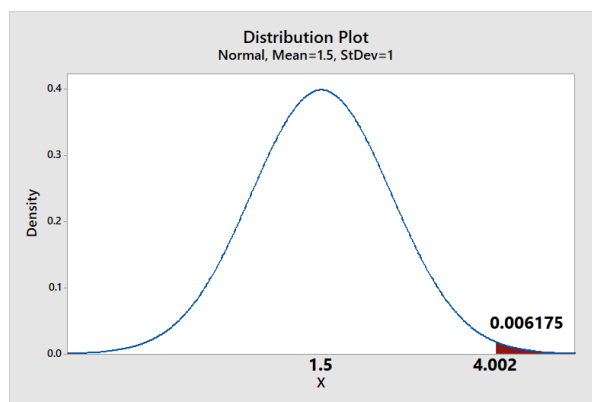
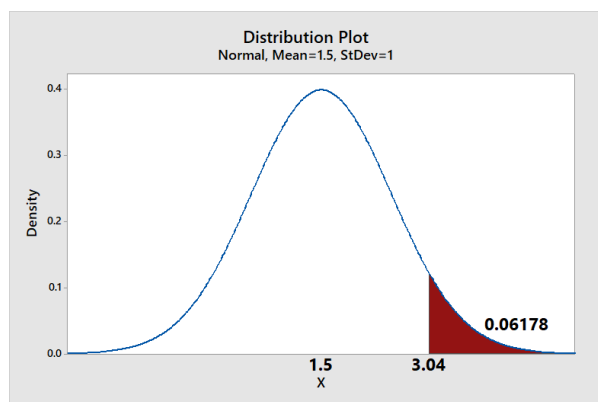
Solution: Determine the short term sigma level that corresponds to 0.0668 area in the tail to the right of this value. Using Minitab, we obtain a 3σ .



- c. Increasing job performance in a corporation from a Short Term Sigma Level of 3.040σ to a Short Term Sigma Level of 4.002σ will cut defects per million by approximately 1/10.

True False

Solution: Short term **3.040σ quality produces ~61780 DPM**, while short term **4.002σ quality produces ~6175 DPM**. Going from 3.04σ to 4.002σ cuts the number of defects per million by 1/10. My wording is confusing, so **I'll also accept 9/10 since the number of defects is reduced by 9/10**, which yields the answer FALSE.



- d. A corporation operating at a Short Term Sigma Level of 3σ has approximately 1,350 DPM.

True **False**

Solution: According to Problem 2(b), a Short Term Sigma Level of 3σ has approximately 66,800 DPM.

3. This problem's information is from the article **US airlines are bumping more travelers as Boeing 737 Max planes grounded** written by Leslie Joseph and appearing online in a June 5, 2019 post by CNBC. The following paragraph is taken directly from this source:

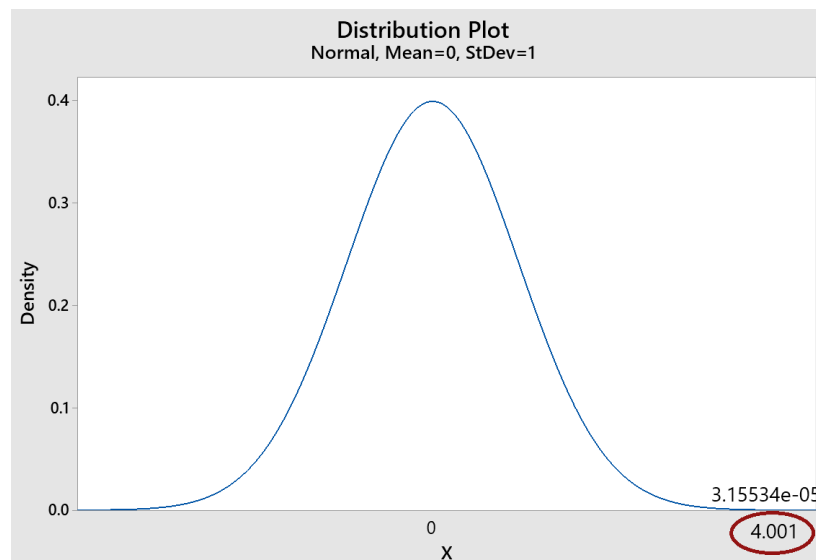
After boasting record low bumping rates, U.S. airlines in the first three months of this year denied boarding to travelers at the highest rate since 2017, according to Department of Transportation data released Wednesday. The increase was partially driven by the grounding of the Boeing 737 Max, which took more than 70 of the high-capacity planes out of service.

In the first quarter, **6,175 passengers were involuntarily denied boarding**. That is nearly triple the number from the same period a year ago but small in comparison with the **195.7 million passengers** who checked in for flights in those three months.

Assuming **no shift** in the process mean over time, determine the **Long Term Sigma Level** associated with the number of bumped passengers out of the 195.7 million passengers.

Use the “**Instructions for submitting your solution**” (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

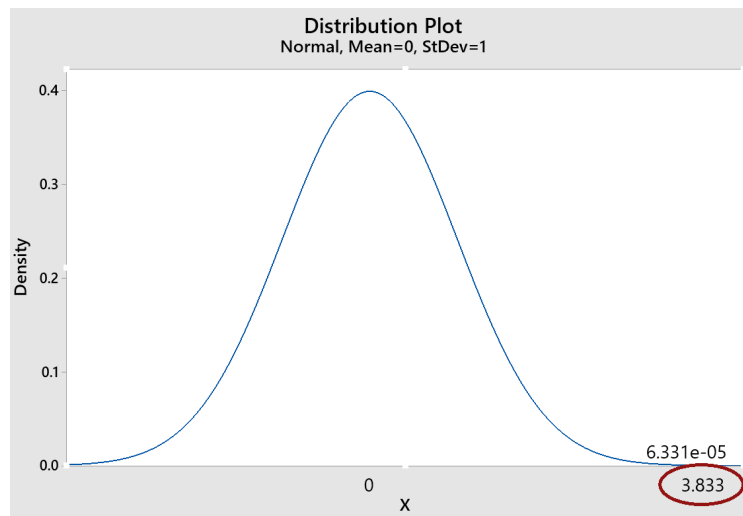
Solution: Since we are told that 6,175 passengers were bumped out of 195.7 million passengers, then the proportion defective is: ~ 0.00003155339806 . Using Minitab's Probability Distribution Plot, we can see that the Sigma Level corresponding to the proportion of defectives is approximately **4.001 σ** .



4. Over the last 18 years, I have literally had over a million pieces of paper printed at the print shop. From hundreds of lectures and assignments with multiple pieces of paper, I have determined that the print shop only has 63.31 defects per million pieces of paper (DPM). The print shop has **NOT** had a shift in the process mean over the past 18 years. What Long Term Sigma Level (with no shift in the process mean) corresponds to their quality service?

Use the “**Instructions for submitting your solution**” (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

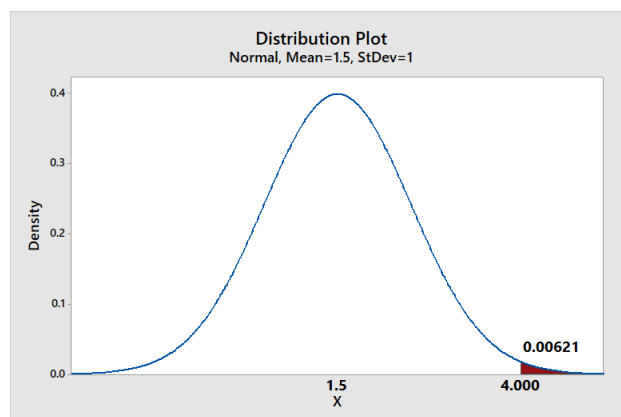
Solution: 63.31 DPM's is associated with 0.00006331 proportion defective in one tail of a normal density curve with mean $\mu = 0$ (long term = no shift in process mean) and standard deviation $\sigma = 1$. Using Minitab, we can determine the corresponding Long Term Sigma Level is approximately **3.833**.



5. If a company is operating at a 4σ Short Term Sigma Level, what proportion of its product is conforming (i.e., non-defective)? Use the “**Instructions for submitting your solution**” (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

Solution: Assume that the random variable X is normally distributed with mean $\mu = 1.5$ (short term = shift) and standard deviation $\sigma = 1$. If a company is operating at a 4σ short term sigma level, then the proportion of its product that is **confirming** is the area to the **LEFT** of $x = 4$. This approximate proportion conforming is $1 - 0.0062 = \mathbf{0.9938}$.

Minitab is used to determine this value, but you could obtain it from a Sigma Level table like the one in the lesson notes.



6. Six Sigma Calculations for a certain company's call response time:

Customer statement: "I consistently wait too long to speak to a customer representative."

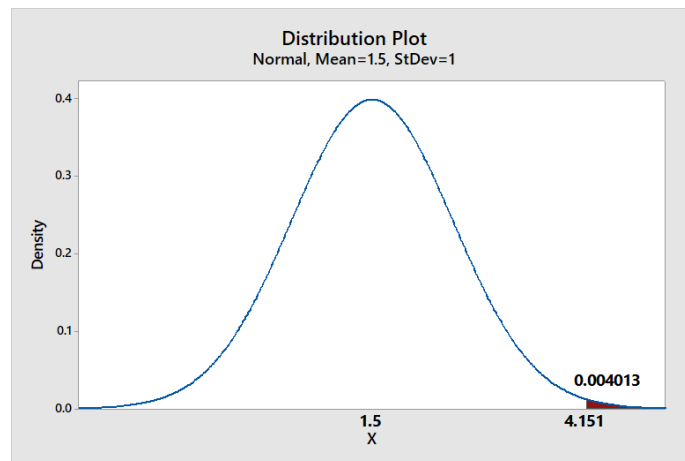
Metric: Time on hold (in seconds) waiting for a customer representative

Defect: Calls with hold times that are at least 60 seconds

Suppose we know that this company's call response time has a Short Term Sigma Level of 4.151σ . How many calls do we expect to be defective out of a million (where defective means being on hold for at least 60 seconds)?

Use the "Instructions for submitting your solution" (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

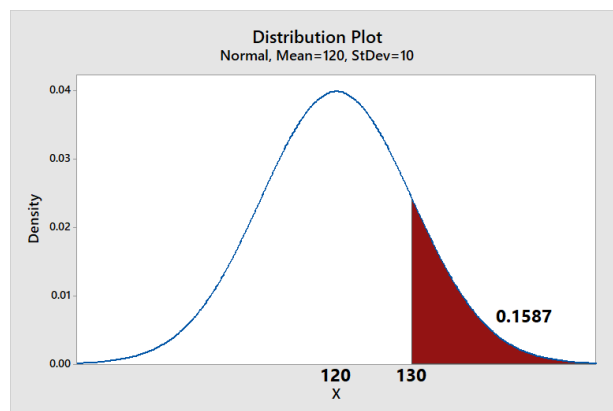
Solution: The area in the right tail of a normal curve with mean $\mu = 1.5$ (short term = shift) and standard deviation $\sigma = 1$ past 4.151 is approximately 0.004013. Thus, we would expect ~**4013** calls out of a million to have a hold time of at least 60 seconds.



7. Suppose that Beyonce, Jay Z, and Solange go bowling together. Each of them has a bowling average X that is normally distributed with mean $\mu = 120$ and standard deviation $\sigma = 10$. Assuming their scores are independent of each other's, determine the following probabilities.

- a. What is the probability that Jay Z's score is more than 130? Give your answer correctly rounded to **3 decimal places**.

Solution: Let X = bowling average for Jay Z (same for Beyonce, Solange); $X \sim \text{Normal}(\mu = 120, \sigma = 10)$. Using Minitab, $P(X > 130) \sim$ **0.1587**.



b. What is the probability that none of the 3 get a score greater than 130? Give your answer correctly rounded to **3 decimal places**.

Solution: Since their scores are independent of each other, then:

$P(X_{JZ} < 130 \text{ AND } X_{Bey} < 130 \text{ AND } X_{Sol} < 130) = P(X_{JZ} < 130) \cdot P(X_{Bey} < 130) \cdot P(X_{Sol} < 130) = (1 - 0.1587)^3$, which is approximately **0.5955**.

What is the probability that at least one of them gets a score greater than 130? Give your answer correctly rounded to **3 decimal places**.

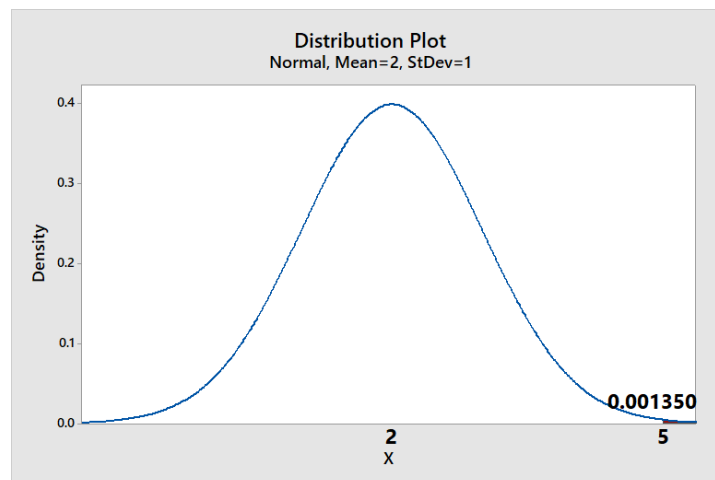
Solution: Suggestion: Use the complement of the event “at least one of them has a score greater than 130.” The complement is “none of them get a score greater than 130.”

$P(X_{JZ} < 130 \text{ OR } X_{Bey} < 130 \text{ OR } X_{Sol} < 130) = 1 - P(\text{all 3 get a score less than 130}) \sim 1 - 0.5955 = \mathbf{0.4045}$.

8. Suppose I know that my company has problems with its process mean drifting from the target by 2σ (instead of the typical short term drift of 1.5σ). How many defects per million would a Short Term Sigma Level of 5σ have given the shift of 2σ (instead of 1.5σ)?

Use the “**Instructions for submitting your solution**” (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

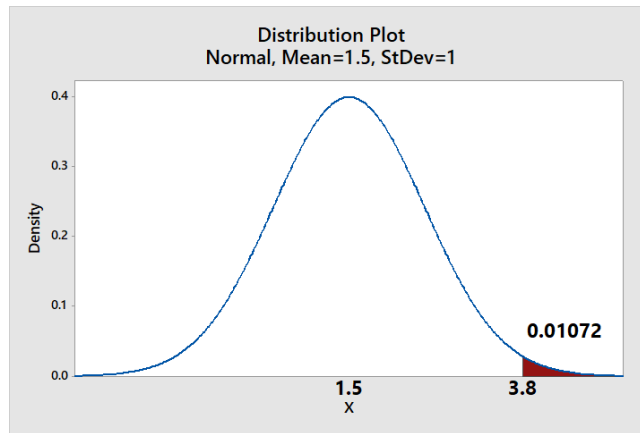
Solution: Assuming the company’s process is normally distributed, let $X \sim \text{Normal}(2, 1)$ – this is a $\text{Normal}(0, 1)$ distribution shifted to the right by 2σ . Then the proportion of x’s in the right tail past 5 is approximately 0.001350 (long term 3σ). Thus, the DPM is **1350**.



9. I have read parts of the book: “**Straight from the Gut**” by the Quality Guru **Jack Welch** (former GE CEO who popularized Six Sigma). He has the following quote on page 334: “A Black Belt team solved the problem and designed a change in the production process that gave the color and static qualities that Sony demanded. We went from **3.8σ** to **5.7σ** and earned Sony’s business.” Assume these are Short Term Sigma Levels and that a shift of 1.5σ occurs.

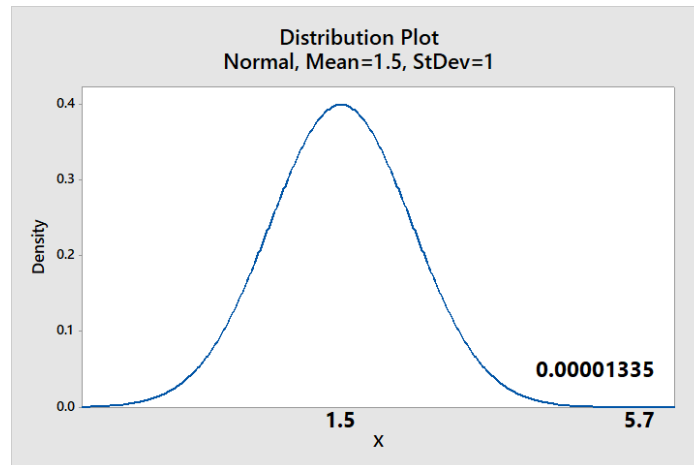
a. As a 3.8σ Short Term Six Sigma company, how many defects per million were they producing? You can round your answer to the nearest integer due to the number of decimal places given in the Minitab output.

Solution: Using Minitab, the approximate number of defects per million was **~ 10720** (rounded). I can get additional decimal places in Minitab using Calc > Probability Distributions.



b. As a 5.7σ Short Term Six Sigma company, how many defects per million were they producing? You can round your answer to the nearest integer since you did this in part (a).

Solution: Using Minitab, the approximate number of defects per million was **~13.35** (rounded).



10. This problem's information is from the article **Fisher-Price Recalls Millions of Rock 'n' Play Sleepers After '30 Baby Deaths'** written by Rebecca Perring and appearing online in an April 15, 2019 post by Express Online. The following paragraph is taken directly from this source:

Fisher-Price toymaker has recalled nearly 5 million of its Rock 'n' Play sleepers after reports 30 babies had died in ten years after using them.

Assuming **no shift** in the process mean over time, determine the **Long Term Sigma Level** associated with 30 of 5 million Rock 'n' Play sleepers being defective.

Use the “**Instructions for submitting your solution**” (e.g. record the answer correctly rounded to 3 decimal places) as displayed with Problem 1.

Solution: The proportion defective of 30 defective sleepers out of 5 million is 0.000006. Using Minitab, this corresponds to a Long Term Sigma Level of **4.378 σ** .

